

# Lifetimes and oscillations of heavy mesons

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## 1 Abstract

We review the theoretical status of the lifetime ratios  $\tau_{B^+}/\tau_{B_d}$  and  $\tau_{B_s}/\tau_{B_d}$  and of the mixing quantities  $\Delta M_s$ ,  $\Delta\Gamma_s$  and  $\phi_s$ . We show that the ratio  $\Delta\Gamma_s/\Delta M_s$  can be determined with almost no non-perturbative uncertainties. Finally we explain how this precise determination of the standard model values can be used to find possible new physics contributions in  $\Delta M_s$ ,  $\Delta\Gamma_s$ ,  $\Delta\Gamma_s/\Delta M_s$  and  $a_{fs}^s$ . Combining the latest experimental bounds on these quantities one already gets some hints for new physics contributions.

## 2 Introduction

Inclusive decays (see e.g. [1] or [2] and references therein) and lifetimes of heavy mesons can be calculated within the framework of the so-called heavy quark expansion (HQE) [3, 4]. In this approach the decay rate is calculated in an expansion in inverse powers of the heavy b-quark mass.

$$\Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2}\Gamma_2 + \frac{\Lambda^3}{m_b^3}\Gamma_3 + \dots \quad (1)$$

$\Gamma_0$  represents the decay of a free heavy b-quark, according to this contribution all b-mesons have the same lifetime. The first correction arises at order  $1/m_b^2$ , they are due to the kinetic and the chromomagnetic operator. At order  $1/m_b^3$  the spectator quark gets involved in the weak annihilation and Pauli interference diagrams [3, 5]. These contributions are numerically enhanced by a phase space factor of  $16\pi^2$ . Each of the  $\Gamma_i$  contains perturbatively calculable Wilson coefficients and non-perturbative parameters, like decay constants or bag parameters. Unfortunately the theoretical predictions for the decay constants vary over a wide range: quenched lattice determinations for  $f_{B_s}$  tend to give values of  $\mathcal{O}(200)$  MeV, while recent unquenched calculations with 2+1 dynamical light flavors give values around 260 MeV - for a

more detailed discussion see [6, 7, 8]. Since lifetime differences depend quadratically on the decay constants, going from 200 MeV to 260 MeV results in an increase of 70%. Here clearly theoretical progress is necessary to pin down the error on the decay constants considerably.

In view of several new theoretical and experimental developments we update the numbers present in the literature (see e.g. [9]).

### 3 Lifetimes

The lifetime ratio of two heavy mesons can be written as

$$\frac{\tau_1}{\tau_2} = 1 + \frac{\Lambda^3}{m_b^3} \left( \Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left( \Gamma_4^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_4^{(1)} + \dots \right) + \dots \quad (2)$$

If one neglects small isospin or SU(3) violating effects one has no  $1/m_b^2$  corrections<sup>1</sup> and a deviation of the lifetime ratio from one starts at order  $1/m_b^3$ .

#### 3.1 $\tau_{B^+}/\tau_{B_d}$

The leading term  $\Gamma_3^{(0)}$  has been determined in [3, 10]. For a quantitative treatment of the lifetime ratios NLO QCD corrections are mandatory -  $\Gamma_3^{(1)}$  has been determined in [11, 12]. Subleading effects of  $\mathcal{O}(1/m_b)$  turned out to be negligible [13]. The matrix elements of the arising four-quark operators have been determined in [14]. Using the result from [11]

$$\begin{aligned} \frac{\tau(B^+)}{\tau(B_d^0)} - 1 &= \tau(B^+) \left[ \Gamma(B_d^0) - \Gamma(B^+) \right] \\ &= 0.0325 \frac{\tau(B^+)}{1.653 \text{ ps}} \left( \frac{|V_{cb}|}{0.04} \right)^2 \left( \frac{m_b}{4.8 \text{ GeV}} \right)^2 \left( \frac{f_B}{200 \text{ MeV}} \right)^2 \\ &\quad \left[ (1.0 \pm 0.2) B_1 + (0.1 \pm 0.1) B_2 - (18.4 \pm 0.9) \epsilon_1 + (4.0 \pm 0.2) \epsilon_2 \right] + \delta_{1/m} \end{aligned} \quad (3)$$

one gets with the matrix elements from Becirevic [14] ( $B_1 = 1.10 \pm 0.20$ ;  $B_2 = 0.79 \pm 0.10$ ;  $\epsilon_1 = -0.02 \pm 0.02$ ;  $\epsilon_2 = 0.03 \pm 0.01$ ) and the values  $V_{cb} = 0.0415$ ,  $m_b = 4.63$  GeV and  $f_B = 216$  MeV [15]:

$$\left[ \frac{\tau(B^+)}{\tau(B_d^0)} \right]_{\text{NLO}} = 1.063 \pm 0.027, \quad (4)$$

which is in excellent agreement with the experimental number [16, 17]

$$\left[ \frac{\tau(B^+)}{\tau(B_d^0)} \right] = 1.071 \pm 0.009. \quad (5)$$

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<sup>1</sup>In the case of  $\tau_{\Lambda_b}/\tau_{B_d}$  these effects are expected to be of the order of 5%.

From Eq. (3) one clearly sees that a precise knowledge of the color octet bag parameters  $\epsilon_1$  and  $\epsilon_2$  - these parameters are of order  $1/N_c$  - is mandatory since their coefficients are numerically enhanced. Here clearly more work has to be done.

### 3.2 $\tau_{B_s}/\tau_{B_d}$

In the lifetime ratio  $\tau_{B_s}/\tau_{B_d}$  a cancellation of weak annihilation contributions arises, that differ only by small SU(3)-violation effects. One expects a number that is close to one [10, 12, 18, 19]

$$\frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01. \quad (6)$$

This expectation is confirmed by experiment [20, 17]

$$\frac{\tau(B_s)}{\tau(B_d)} = 0.957 \pm 0.027, \quad (7)$$

although more precise experimental numbers would be very desirable.

### 3.3 $\tau_{B_c^+}$

The lifetime of the doubly heavy meson  $B_c$  has been investigated in [21]

$$\tau(B_c) = 0.52^{+0.18}_{-0.12} \text{ ps}. \quad (8)$$

In addition to the b-quark now also the c-charm quark can decay, giving rise to the biggest contribution to the total decay rate. The current experimental number [22]

$$\tau(B_c) = 0.469 \pm 0.027 \text{ ps} \quad (9)$$

agrees nicely with the theoretical prediction, but it has much smaller errors. Here clearly some theoretical improvements are necessary to pin down the error.

## 4 Mixing Parameters

In this section we briefly investigate the status of the mixing parameters. For a more detailed review we refer the interested reader to [7].

The mixing of the neutral B-mesons is described by the off diagonal elements  $\Gamma_{12}$  and  $M_{12}$  of the mixing matrix.  $\Gamma_{12}$  stems from the absorptive part of the box diagrams - only internal up and charm quarks contribute, while  $M_{12}$  stems from the dispersive part of the box diagram, therefore being sensitive to heavy internal particles like the top quark or heavy new physics particles. By diagonalizing the mixing matrix we

obtain the physical eigenstates  $B_H$  and  $B_L$  with defined masses ( $M_H, M_L$ ) and defined decay rates ( $\Gamma_H, \Gamma_L$ ) in terms of the flavor eigenstates  $B_s = (\bar{b}s)$  and  $\bar{B}_s = (b\bar{s})$ :

$$B_H := p B + q \bar{B}, \quad B_L := p B - q \bar{B} \quad \text{with } |p|^2 + |q|^2 = 1. \quad (10)$$

The calculable quantities  $|M_{12}|$ ,  $|\Gamma_{12}|$  and  $\phi = \arg(-M_{12}/\Gamma_{12})$  can be related to three observables:

- **Mass difference:**

$$\Delta M := M_H - M_L = 2|M_{12}| \left( 1 + \frac{1}{8} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} \sin^2 \phi + \dots \right) \quad (11)$$

$|M_{12}|$  is due to heavy internal particles in the boxdiagrams like the top-quark or SUSY-particles.

- **Decay rate difference:**

$$\Delta \Gamma := \Gamma_L - \Gamma_H = 2|\Gamma_{12}| \cos \phi \left( 1 - \frac{1}{8} \frac{|\Gamma_{12}|^2}{|M_{12}|^2} \sin^2 \phi + \dots \right) \quad (12)$$

$|\Gamma_{12}|$  is due to light internal particles: particles, like the up- and the charm-quark. It is therefore very insensitive to new physics contributions.

- **Flavor specific/semileptonic CP asymmetries:**

A decay  $B_q \rightarrow f$  is called flavor specific, if the decays  $\bar{B}_q \rightarrow f$  and  $B_q \rightarrow \bar{f}$  are forbidden and if no direct CP violation occurs, i.e.  $|\langle f|B_q \rangle| = |\langle \bar{f}|\bar{B}_q \rangle|$ . Some examples are  $B_s \rightarrow D_s^- \pi^+$  or  $B_q \rightarrow X l \nu$  (therefore the name semileptonic CP asymmetry). The flavor specific CP asymmetry is defined as

$$\begin{aligned} a_{fs} &= \frac{\Gamma(\bar{B}_q(t) \rightarrow f) - \Gamma(B_q(t) \rightarrow \bar{f})}{\Gamma(\bar{B}_q(t) \rightarrow f) + \Gamma(B_q(t) \rightarrow \bar{f})} = -2 \left( \left| \frac{q}{p} \right| - 1 \right) \\ &= \text{Im} \frac{\Gamma_{12}}{M_{12}} = \frac{\Delta \Gamma}{\Delta M} \tan \phi. \end{aligned}$$

## 4.1 Mass difference

Calculating the box diagram with internal top quarks one obtains

$$M_{12,q} = \frac{G_F^2}{12\pi^2} (V_{tq}^* V_{tb})^2 M_W^2 S_0(x_t) B_{B_q} f_{B_q}^2 M_{B_q} \hat{\eta}_B \quad (13)$$

The Inami-Lim function  $S_0(x_t = \bar{m}_t^2/M_W^2)$  [23] is the result of the box diagram without any gluon corrections. The NLO QCD correction is parameterized by  $\hat{\eta}_B \approx 0.84$

[24]. The non-perturbative matrix element of the operator  $Q = (\bar{b}q)_{V-A}(\bar{b}q)_{V-A}$  is parameterized by the bag parameter  $B$  and the decay constant  $f_B$

$$\langle \overline{B_q} | Q | B_q \rangle = \frac{8}{3} B_{B_q} f_{B_q}^2 M_{B_q}. \quad (14)$$

Using the conservative estimate  $f_{B_s} = 240 \pm 40$  MeV [7] and the bag parameter  $B$  from JLQCD [25] we obtain

$$\Delta M_s = 19.3 \pm 6.4 \pm 1.9 \text{ ps}^{-1} \quad (15)$$

The first error stems from the uncertainty in  $f_{B_s}$  and the second error summarizes the remaining theoretical uncertainties. The determination of  $\Delta M_d$  is affected by even larger uncertainties because here one has to extrapolate to the small mass of the down-quark. The ratio  $\Delta M_s / \Delta M_d$  is theoretically better under control since in the ratio of the non-perturbative parameters many systematic errors cancel.

This year also  $\Delta M_s$  was measured, leading to the pleasant situation of having very precise experimental numbers at hand [20, 26, 27]

$$\Delta M_d = 0.507 \pm 0.004 \text{ ps}^{-1}, \quad (16)$$

$$\Delta M_s = 17.77 \pm 0.12 \text{ ps}^{-1}. \quad (17)$$

To be able to distinguish possible new physics contributions to  $\Delta M_s$  from QCD uncertainties much more precise numbers for  $f_{B_s}$  are needed.

## 4.2 Decay rate difference and flavor specific CP asymmetries

In order to determine the decay rate difference of the neutral B-mesons and flavor specific CP asymmetries a precise determination of  $\Gamma_{12}$  is needed. With the help of the HQE  $\Gamma_{12}$  can be written as

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3} \left( \Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \dots \right) + \frac{\Lambda^4}{m_b^4} \left( \Gamma_4^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_4^{(1)} + \dots \right) + \dots \quad (18)$$

The leading term  $\Gamma_3^{(0)}$  was determined in [28]. The numerical and conceptual important NLO-QCD corrections ( $\Gamma_3^{(1)}$ ) were determined in [29, 30, 31]. Subleading  $1/m$ -corrections, i.e.  $\Gamma_4^{(0)}$  were calculated in [19, 32] and even the Wilson coefficients of the  $1/m^2$ -corrections ( $\Gamma_5^{(0)}$ ) were calculated and found to be small [33].

Besides the already known operator  $Q$  in the calculation of  $\Gamma_{12}$  three additional operators arise  $\tilde{Q}$ ,  $Q_S$  and  $\tilde{Q}_S$ . The tilde stands for a color rearrangement and index  $S$  corresponds to a S-P Dirac structure instead of the V-A-structure.  $Q = \tilde{Q}$  and it

can be shown [19, 29, 7] that a certain combination of  $Q, Q_S$  and  $\tilde{Q}_S$  is suppressed by powers of  $1/m_b$  and  $\alpha_s$

$$\tilde{Q} = Q \quad \text{and} \quad R_0 = Q_S + \alpha_1 \tilde{Q}_S + \frac{\alpha_2}{2} Q = \mathcal{O}\left(\frac{1}{m_b}, \alpha_s\right) \quad (19)$$

with  $\alpha_i = 1 + \mathcal{O}(\alpha_s)$ , for more details see [7]. In the literature [28, 19, 29, 30, 31] always  $\tilde{Q}_S$  was eliminated - with the help of Eq. (19) - and one was left with the operator basis  $\{Q, Q_S\}$ , which we call in the following the *old basis*. Working in the old basis one finds several serious drawbacks:

- An almost complete cancellation of the coefficient of the operator  $Q$  takes place, while the operator  $Q_S$  is dominant. So in the ratio  $\Delta\Gamma_s/\Delta M_s$  the only coefficient that is free of non-perturbative uncertainties is numerically negligible.
- The  $1/m$  corrections are abnormally large - all contributions have the same sign.
- The  $\alpha_s$ -corrections and the remaining  $\mu$ -dependence is unexpectedly large.

In [7] it was found, that expressing  $\Gamma_{12}$  in terms of the *new basis*  $\{Q, \tilde{Q}_S\}$  one gets a result, that is free of the above shortcomings. The change of the basis corresponds to throwing away certain contributions of  $\mathcal{O}(\alpha_s^2)$  and  $\mathcal{O}(\alpha_s/m_b)$ , which is beyond the calculated accuracy. For our new determination of  $\Gamma_{12}$  we also use the  $\overline{\text{MS}}$ -scheme [34], besides the pole scheme for the b-quark mass. Moreover we sum up logarithms of the form  $z \ln z$  - with  $z = m_c^2/m_b^2$  - to all orders, following [11] and of course we have to include also subleading CKM-structures to determine  $a_{fs}$ , as done in [30, 31].

In the old basis one obtains

$$\Delta\Gamma_s = \left(\frac{f_{B_s}}{240 \text{ MeV}}\right)^2 \left[0.002B + 0.094B'_S - \left(0.033B_{\tilde{R}_2} + 0.019B_{R_0} + 0.005B_R\right)\right], \quad (20)$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = 10^{-4} \cdot \left[0.9 + 40.9\frac{B'_S}{B} - \left(14.4\frac{B_{\tilde{R}_2}}{B} + 8.5\frac{B_{R_0}}{B} + 2.1\frac{B_R}{B}\right)\right], \quad (21)$$

with

$$\langle \overline{B_s} | Q_S | B_s \rangle = -\frac{5}{3} B'_S f_{B_s}^2 M_{B_s}, \quad B'_X := B_X \frac{M_{B_s}^2}{(\overline{m}_b + \overline{m}_s)^2}. \quad (22)$$

In Eq. (21) we have explicitly shown the dependence on the dominant  $1/m$  operators  $R_2$  and  $R_0$  (see [19, 7] for the definition). The remaining power corrections are summarized in the coefficient of  $B_R$ . One clearly sees that the cancellation in the coefficient of  $B$  leads to the undesirable situation, that the only coefficient in  $\Delta\Gamma/\Delta M$  that is free of non-perturbative uncertainties is negligible.

This changes however dramatically if one uses the new basis

$$\Delta\Gamma_s = \left(\frac{f_{B_s}}{240\text{MeV}}\right)^2 \left[0.105B + 0.024\tilde{B}'_S - \left(0.030B_{\tilde{R}_2} - 0.006B_{R_0} + 0.003B_R\right)\right] \quad (23)$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = 10^{-4} \cdot \left[46.2 + 10.6\frac{B'_S}{B} - \left(13.2\frac{B_{\tilde{R}_2}}{B} - 2.5\frac{B_{R_0}}{B} + 1.2\frac{B_R}{B}\right)\right] \quad (24)$$

with

$$\langle \overline{B}_s | \tilde{Q}_S | B_s \rangle = \frac{1}{3} \tilde{B}'_S f_{B_s}^2 M_{B_s}. \quad (25)$$

Now the dominant part of  $\Delta\Gamma/\Delta M$  can be determined without any hadronic uncertainties!

Using the non-perturbative parameters from [25, 35], we obtain the following final numbers (see [7] for the complete list of the numerical input parameters)

$$\Delta\Gamma_s = (0.096 \pm 0.039) \text{ ps}^{-1} \Rightarrow \frac{\Delta\Gamma_s}{\Gamma_s} = \Delta\Gamma_s \cdot \tau_{B_d} = 0.147 \pm 0.060, \quad (26)$$

$$a_{f_s}^s = (2.06 \pm 0.57) \cdot 10^{-5}, \quad (27)$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = (49.7 \pm 9.4) \cdot 10^{-4}, \quad (28)$$

$$\phi_s = 0.0041 \pm 0.0008 = 0.24^\circ \pm 0.04. \quad (29)$$

The composition of the theoretical error of  $\Delta\Gamma$  is compared for the use of the old and the new basis in Fig. (1). The by far dominant error comes from the decay constant  $f_{B_s}$ , followed by the uncertainty due to the power suppressed operator  $\tilde{R}_2$  and the remaining  $\mu$ -dependence. In this case the theoretical improvement due to the change of basis is somehow limited by the huge uncertainty due to  $f_{B_s}$ , which is the same in both bases.

This changes if one looks at the composition of the theoretical error of  $\Delta\Gamma/\Delta M$  in Fig. (2). Since now  $f_{B_s}$  cancels the dominant error comes from the uncertainty due to the power suppressed operator  $\tilde{R}_2$  and the remaining  $\mu$ -dependence.

One clearly sees that the change of the basis resulted in a considerable reduction of the theoretical error, almost a factor 3 in the case of  $\Delta\Gamma_s/\Delta M_s$ !

To improve our theoretical knowledge of the mixing quantities further one needs more precise values of the non-perturbative parameters, like the decay constants or the power suppressed operators. If accurate non-perturbative parameters are available one might think about NNLO calculations ( $\alpha_s^2$  or  $\alpha_s/m_b$ -corrections) to reduce the remaining  $\mu$ -dependence.

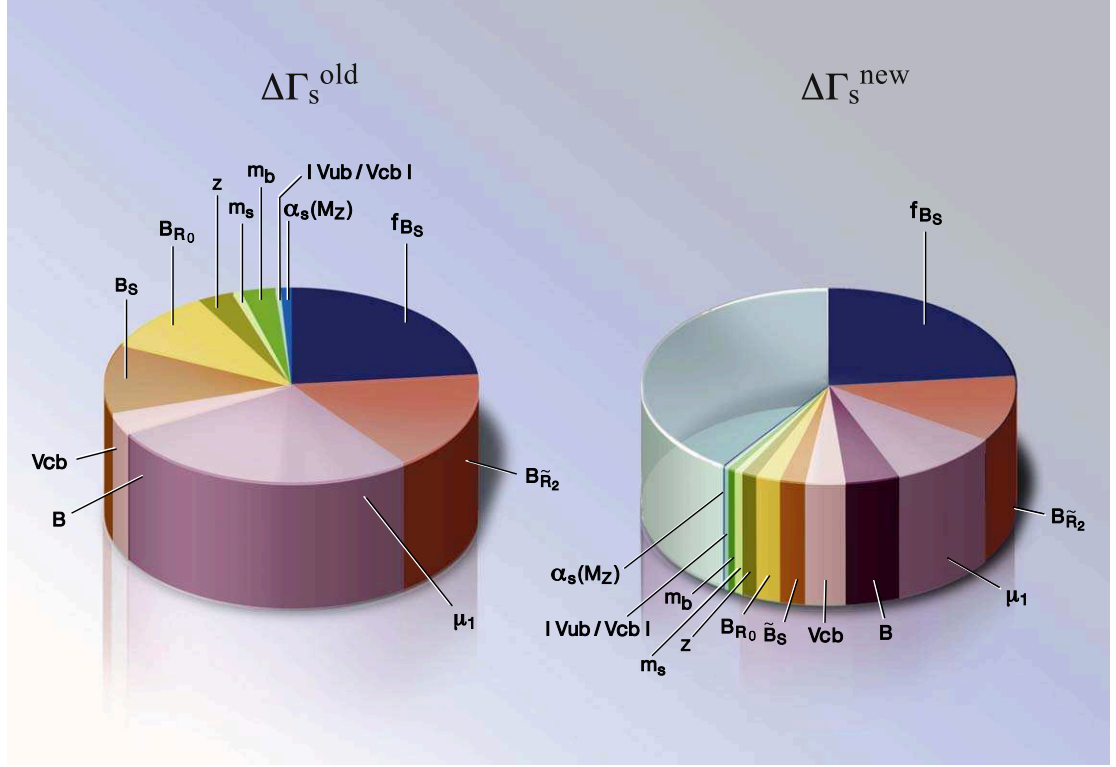


Figure 1: Error budget for the determination of  $\Delta\Gamma_s$  in the old and the new basis.

### 4.3 New Physics

New physics (see e.g. [36]) is expected to have almost no impact on  $\Gamma_{12}$  but it can change  $M_{12}$  considerably. Therefore one can write

$$\Gamma_{12,s} = \Gamma_{12,s}^{\text{SM}}, \quad M_{12,s} = M_{12,s}^{\text{SM}} \cdot \Delta_s; \quad \Delta_s = |\Delta_s| e^{i\phi_s^\Delta} \quad (30)$$

With this parameterisation the physical mixing parameters can be written as

$$\Delta M_s = 2|M_{12,s}^{\text{SM}}| \cdot |\Delta_s| \quad (31)$$

$$\Delta\Gamma_s = 2|\Gamma_{12,s}| \cdot \cos(\phi_s^{\text{SM}} + \phi_s^\Delta) \quad (32)$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\text{SM}}|} \cdot \frac{\cos(\phi_s^{\text{SM}} + \phi_s^\Delta)}{|\Delta_s|} \quad (33)$$

$$a_{fs}^s = \frac{|\Gamma_{12,s}|}{|M_{12,s}^{\text{SM}}|} \cdot \frac{\sin(\phi_s^{\text{SM}} + \phi_s^\Delta)}{|\Delta_s|} \quad (34)$$

Now we combine the current experimental knowledge about the mixing parameters to find out whether  $B_s$ -mixing is described by the standard model alone, or whether



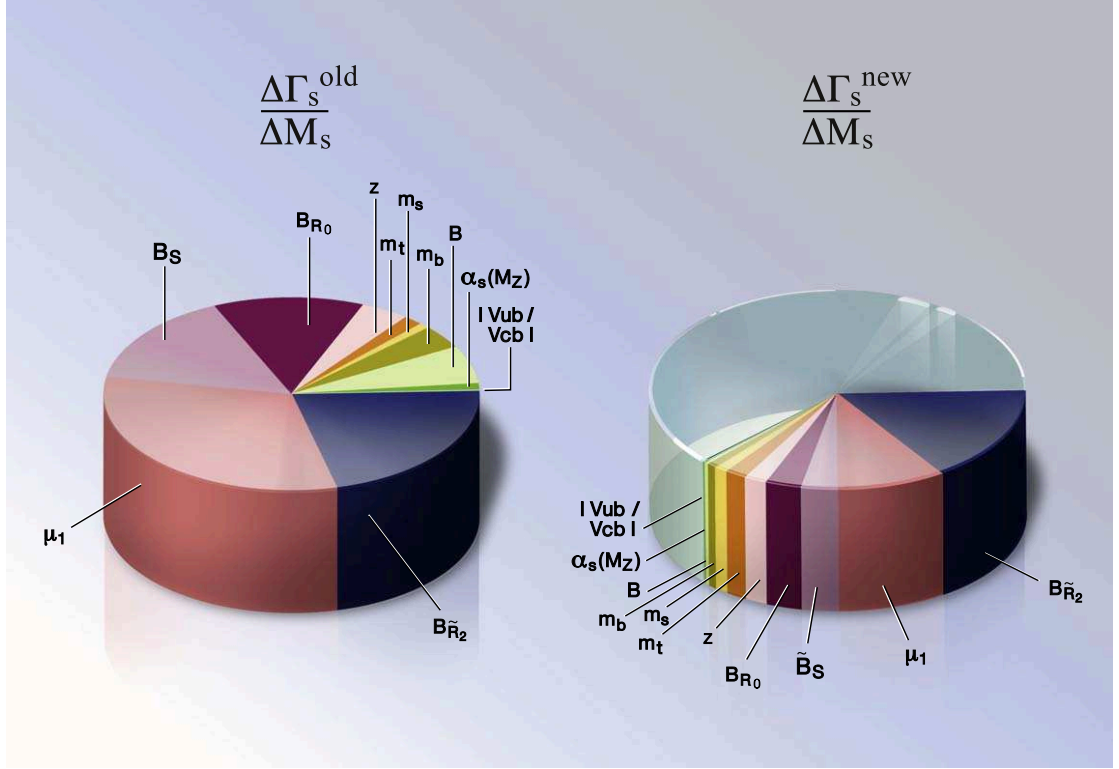


Figure 2: Error budget for the determination of  $\Delta\Gamma_s/\Delta M_s$  in the old and the new basis.

we already get some signals of new physics contributions.

The mass difference  $\Delta M_s$  is now known very precisely [26, 27]

$$\Delta M_s = 17.77 \pm 0.10_{(\text{syst})} \pm 0.07_{(\text{stat})} \text{ ps}^{-1} \quad \text{CDF.} \quad (35)$$

For the remaining mixing parameters in the  $B_s$ -system only experimental bounds are available. The width difference  $\Delta\Gamma_s/\Gamma_s$  was investigated at ALEPH, BELLE, CDF, D0 by analyzing the decays  $B_s \rightarrow D_s^{(*)+} + D_s^{(*)-}$  [37],  $B_s \rightarrow J/\Psi + \phi$  [38] and  $B_s \rightarrow K^+ + K^-$  [39] and the flavor specific lifetime of the  $B_s$  meson [40]. A recent combination of all these results yields [41]

$$\Delta\Gamma_s = 0.097 \pm 0.042 \text{ ps}^{-1}, \quad (36)$$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = (56 \pm 24) \times 10^{-4}. \quad (37)$$

Except the angular analysis  $B_s \rightarrow J/\Psi + \phi$  all other determinations are affected by some drawbacks, described in [7]. Moreover the D0 collaboration has updated their

results [38] for the decay  $B_s \rightarrow J/\Psi + \phi$  in [42, 17, 26] using  $1\text{fb}^{-1}$  of data. Setting the value of the mixing phase  $\phi_s$  to zero they obtain [42, 17, 26]

$$\Delta\Gamma_s = 0.12 \pm 0.08^{+0.03}_{-0.04} \text{ps}^{-1}, \quad (38)$$

allowing for a non-zero value of the mixing phase  $\phi_s$  they get

$$\Delta\Gamma_s = 0.17 \pm 0.09 \pm 0.03 \text{ps}^{-1}, \quad (39)$$

$$\phi_s = -0.79 \pm 0.56 \pm 0.01. \quad (40)$$

In the following we will for  $\Delta\Gamma_s$  and  $\phi_s$  only use the numbers from Eq. (39) and Eq. (40).

The semileptonic CP asymmetry in the  $B_s$  system has been determined directly in [43] and found to be

$$a_{sl}^{s,\text{direct}} = (24.5 \pm 19.3 \pm 3.5) \cdot 10^{-3}. \quad (41)$$

Moreover the semileptonic CP asymmetry can be extracted from the same sign dimuon asymmetry that was measured in [44] to be

$$a_{sl} = (-2.8 \pm 1.3 \pm 0.9) \cdot 10^{-3}. \quad (42)$$

Updating the numbers in [45, 46] one sees that

$$a_{sl} = (0.582 \pm 0.030) a_{sl}^d + (0.418 \pm 0.047) a_{sl}^s \quad (43)$$

In [45, 46] the experimental bound for  $a_{sl}^d$  was used to extract from Eq.(42) and Eq.(43) a bound on  $a_{sl}^s$ . Due to the huge experimental uncertainties in  $a_{sl}^d$  this strategy resulted in a large error on  $a_{sl}^s$ . Since in the  $B_d$ -system there is not much room left for new physics contributions, we think it is justified to use the theoretical number of  $a_{sl}^d$ . Using  $a_{sl}^d = -(0.48 \pm 0.12) \cdot 10^{-3}$  we get from Eq.(42), Eq.(43) and Eq.(42) already a nice bound

$$a_{sl}^{s,\text{dimuon}} = (-6.0 \pm 3.2 \pm 2.2) \cdot 10^{-3}. \quad (44)$$

Combining this number with the direct determination [43] we get our final experimental number for the semileptonic CP asymmetries

$$a_{sl}^s = (-5.2 \pm 3.2 \pm 2.2) \cdot 10^{-3}. \quad (45)$$

Now we combine these experimental numbers with the theoretical errors to extract bounds in the imaginary  $\Delta_s$ -plane by the use of Eqs. (31), (32), (33) and (34), see Fig. (3).

The comparison of experiment and standard model expectation for  $\Delta M_s$ ,  $\Delta\Gamma_s$ ,  $\phi_s$ ,  $\Delta\Gamma_s/\Delta M_s$  and  $a_{sl}^s$  presented in figure 3 already shows some hints for deviations from the standard model.

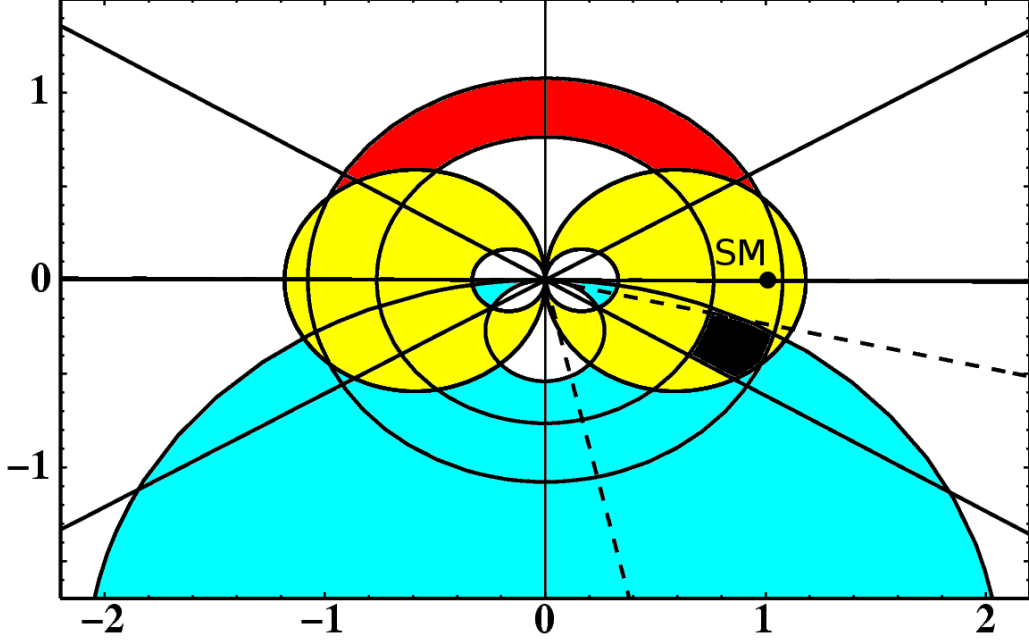


Figure 3: Current experimental bounds in the complex  $\Delta_s$ -plane. The bound from  $\Delta M_s$  is given by the red (dark-grey) ring around the origin. The bound from  $\Delta\Gamma_s/\Delta M_s$  is given by the yellow (light-grey) region and the bound from  $a_{fs}^s$  is given by the light-blue (grey) region. The angle  $\phi_s^\Delta$  can be extracted from  $\Delta\Gamma_s$  (solid lines) with a four fold ambiguity - one bound coincides with the x-axis! - or from the angular analysis in  $B_s \rightarrow J/\Psi\phi$  (dashed line). If the standard model is valid all bounds should coincide in the point (1,0). The current experimental situation shows a small deviation, which might become significant, if the experimental uncertainties in  $\Delta\Gamma_s$ ,  $a_{sl}^s$  and  $\phi_s$  will go down in near future.

## 5 Conclusion and outlook

Theoretical predictions of the lifetimes of heavy mesons are in excellent agreement with the experimental numbers. We do not see any signal of possible duality violations. To become more quantitative in the prediction of  $\tau_{B^+}/\tau_{B_d}$  the non-perturbative estimates of the bag parameters  $B_1$ ,  $B_2$  and  $\epsilon_1$ ,  $\epsilon_2$  have to be improved. For  $\tau_{B_s}/\tau_{B_d}$  more precise experimental numbers are needed, while in the case of  $\tau_{B_c}$  theoretical progress is mandatory.

The theoretical uncertainty in the mixing parameter  $\Delta M$  is completely dominated by the decay constant. We have presented a method (see [7] for more details) to reduce the theoretical error in  $\Delta\Gamma$ ,  $\Delta\Gamma/\Delta M$  and  $a_{fs}$  considerably. This relatively clean standard model predictions can be used to look for new physics effects in  $B_s$ -mixing. From the currently available experimental bounds on  $\Delta\Gamma_s$  and  $a_{fs}$  one already gets some hints for deviations from the standard model. This situation will improve dramatically as soon as more data are available.

For a further reduction of the theoretical uncertainty in  $\Delta\Gamma$  a much higher accuracy than currently available on the decay constants is necessary. If this problem is solved or if one looks at quantities like  $\Delta\Gamma/\Delta M$  and  $a_{fs}$  where the dependence on  $f_B$  cancels, then the dominant uncertainty comes from the unknown matrix elements of the power suppressed operators. Here any non-perturbative estimate would be very desirable. If accurate non-perturbative parameters are available one might think about NNLO calculations ( $\alpha_s^2$  or  $\alpha_s/m_b$ -corrections) to reduce the remaining  $\mu$ -dependence.

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